

TD, week 7: Axiomatic semantics

Ex. 1 : Warmup

- Write down carefully the full derivation of

$$\{x = n \wedge y = m\} z \leftarrow x; x \leftarrow y; y \leftarrow z \{x = m \wedge y = n\}$$

according to the rules of HOARE Logic

- What is the meaning of:
 1. $\vdash \{\perp\} c \{A\}$?
 2. $\vdash \{A\} c \{\top\}$?
 3. $\vdash \{\top\} c \{A\}$?
 4. $\vdash \{A\} c \{\perp\}$?

Ex. 2 : DIJKSTRA's Dutch National Flag

Suppose that memory locations can take colors Red, White and Blue as values (they can be encoded with numbers, if you prefer). Given an array \mathbf{a} , you can express the length $\text{length}(\mathbf{a})$ of \mathbf{a} and for each $i \leq \text{length}(\mathbf{a})$ you can refer to the i^{th} memory location with $\mathbf{a}[i]$.

1. Write a program DNF which sorts an array of memory locations in such a way that it resembles the Dutch national flag (ie. it is sorted wrt. the order Red < White < Blue).
2. Formalize the specification in terms of pre- and post-conditions A and B . Argue that $\vdash \{A\} \text{DNF} \{B\}$ holds.
3. In order to judge how good is your invariant A , think of the following "buggy" implementation DNF' of the "Dutch National Flag" specification: given the input array x_1, \dots, x_n , behave as DNF if $n < 3$, otherwise write Red in x_1 , White in x_2 and Blue in x_3, \dots, x_n . Does it satisfy A at the beginning and B at the end of the execution? Can you think of different A s and B s which rule out DNF', i.e. such that $\not\vdash \{A\} \text{DNF}' \{B\}$?

This should make you reflect on expressiveness of HOARE Logic.

Ex. 3 : Exceptions

Consider the extension of the language with two constructions: `raise α` and `trap α in C_1 with C_2` .

1. Explain how to extend the denotational semantics by continuation to these constructs.
2. Deduce new rules for the axiomatic semantics.
3. Prove the correctness of this new system.