

# Math exam

## 1 Propositional logic

Let  $\psi$  be the formula  $((A \vee (\neg B)) \Rightarrow B)$ .

1. Write the truth table of  $\psi$ .
2. Is  $\psi$  a tautology?
3. Is  $\psi$  a contradiction?

## 2 Induction

Let

$$f : \mathbb{R}^+ \rightarrow \mathbb{R}$$

$$x \mapsto \frac{x}{x+1}$$

For  $n \in \mathbb{N}^*$ , we denote

$$f^{(n)}(x) = \begin{cases} f(x) & \text{if } n = 1 \\ f \circ f^{(n-1)}(x) & \text{otherwise} \end{cases}$$

1. Show that the predicate  $P_n : "\forall x \in \mathbb{R}^+, f^{(n)}(x) = \frac{x}{nx+1}"$  holds over  $\mathbb{N}^*$

## 3 Relations

Let  $\mathfrak{R}$  be the relation defined over  $\mathbb{R}^2$  by

$$(a, b) \mathfrak{R} (c, d) \text{ ssi } a + d = c + b$$

1. Show that  $\mathfrak{R}$  is an equivalence relation over  $\mathbb{R}^2$ .
2. Find the equivalence classes of  $(0, 1)$ ,  $(0, 0)$  and  $(1, -1)$ . Draw these sets in the  $\mathbb{R}^2$  plane.

## 4 Functions

1. Let  $c \in \mathbb{R}$ . Solve for  $X \in \mathbb{R}^{+*}$  the equation

$$X^2 - 2cX - 1 = 0$$

2. Solve over  $\mathbb{R}$  the equation parametrized by  $c \in \mathbb{R}$

$$\frac{e^x - e^{-x}}{2} = c$$

We could let  $X = e^x$ .

3. Let  $\sinh$  be the hyperbolic sine function defined by

$$\begin{aligned} \sinh : \mathbb{R} &\rightarrow \mathbb{R} \\ x &\mapsto \frac{e^x - e^{-x}}{2} \end{aligned}$$

Is it injective, surjective and/or bijective ?