TD1: propositional logic (LP_0)

Exercise 1: Some tables

Find formulas of A et B matching the following tables:

$[A]_\sigma$	$[B]_\sigma$	$[\varphi_1]_\sigma$	$[\varphi_2]_\sigma$	$[\varphi_3]_\sigma$	$[\varphi_4]_\sigma$	$[\varphi_5]_\sigma$	$[\varphi_6]_\sigma$
ff	ff	ff	tt	ff	tt	tt	ff
ff	tt	ff	ff	tt	tt	tt	tt
tt	ff	ff	ff	tt	ff	tt	ff
tt	tt	tt	ff	ff	tt	tt	tt

Solution: Solutions are not unique. These are simple solutions.

- $\varphi_1 = (A \wedge B)$
- $\varphi_2 = (\neg(A \vee B))$
- $\varphi_3 = (\neg(A \Leftrightarrow B))$
- $\varphi_4 = (A \Rightarrow B)$
- $\varphi_5 = \top$
- $\varphi_6 = B$

Exercise 2: Some formulas

Write the tables of

(a) $(\neg A)$

(b) $(\neg(A \wedge B))$

(c) $((A \wedge B) \vee ((\neg A) \wedge (\neg B)))$

Solution:

$[A]_\sigma$	$[B]_\sigma$	$[(\neg A)]_\sigma$	$[(\neg(A \wedge B))]_\sigma$	$[((A \wedge B) \vee ((\neg A) \wedge (\neg B)))]_\sigma$
ff	ff	tt	tt	tt
ff	tt	tt	tt	ff
tt	ff	ff	tt	ff
tt	tt	ff	ff	tt

Exercise 3: Natural language

Are the following propositions true or not?

(a) The fact that Napoléon is dead imply that he won the battle of Waterloo.

(b) The fact that a mathematic professor of yours is the queen of England imply that one of your biology professors is the king of Spain.

(c) The fact that I will win the lottery at least once in my whole life imply that water wets.

Solution:

(a) A = « Napoléon is dead » ; B = « Napoléon won the battle of Waterloo » A is true (Napoléon died the 5th of May 1821 on Saint Helena Island in the Longwood House) ; B is false (Napoléon lost the battle of Waterloo the 18th of June 1815 against perfidious Albion, Ireland, Prussia, Netherlands and some little known duchies and kingdoms) $(A \Rightarrow B)$ is false. So the proposition is false.

(b) A = « A mathematic professor of yours is the queen of England » ; B = « One of your biology professors is the king of Spain ». A and B are (probably) false. So $(A \Rightarrow B)$ is true.

(c) A = « I will win the lottery at least once in my whole life » ; B = « Water wets ». A is probably false, but can be true. We will assume that both cases are possible. B is true. $(A \Rightarrow B)$ is true whether A is true or not. So the proposition is true, even for the more pessimists of you.

Exercise 4:

An inspector of public health services is inspecting a psychiatric hospital where some strange cases were reported. In this hospital, there are only patients and physicians, but both of them can be sound of mind or totally crazy. The inspector has to take out people that have nothing to do here, that is sane patients and crazy doctors (at the risk of reintegrate them later as patients). He assumes that sane persons tell only the truth, while insane persons say only false things. In a room, he interviews two persons (named A and B to preserve their anonymity). A says that B is insane and B says that A is a doctor.

After an long thinking, the inspector bring out one of the two of the hospital. Which one (and why?)

Is there anything to say about the other?

Solution:

- α : A is a physician
- β : A is insane
- γ : B is a physician
- δ : B in insane
- $\varepsilon = (\alpha \Leftrightarrow \beta)$: A have to leave
- $\zeta = (\gamma \Leftrightarrow \delta)$: B have to leave
- $\eta = (\neg(\beta \Leftrightarrow \delta))$: A says that B is insane
- $\theta = (\neg(\alpha \Leftrightarrow \delta))$: B says that A is a physician

We know that η et θ are true.

$[\alpha]_\sigma$	$[\beta]_\sigma$	$[\gamma]_\sigma$	$[\delta]_\sigma$	$[\varepsilon]_\sigma$	$[\zeta]_\sigma$	$[\eta]_\sigma$	$[\theta]_\sigma$
ff	*ff*	*ff*	*ff*	*tt*	*tt*	*ff*	*ff*
ff	*ff*	*ff*	*tt*	*tt*	*ff*	*tt*	*tt*
ff	*ff*	*tt*	*ff*	*tt*	*ff*	*ff*	*ff*
ff	*ff*	*tt*	*tt*	*tt*	*tt*	*tt*	*tt*
ff	*tt*	*ff*	*ff*	*ff*	*tt*	*tt*	*ff*
ff	*tt*	*ff*	*tt*	*ff*	*ff*	*ff*	*tt*
ff	*tt*	*tt*	*ff*	*ff*	*ff*	*tt*	*ff*
ff	*tt*	*tt*	*tt*	*ff*	*tt*	*ff*	*tt*
tt	*ff*	*ff*	*ff*	*ff*	*tt*	*ff*	*tt*
tt	*ff*	*ff*	*tt*	*ff*	*ff*	*tt*	*ff*
tt	*ff*	*tt*	*ff*	*ff*	*ff*	*ff*	*tt*
tt	*ff*	*tt*	*tt*	*ff*	*tt*	*tt*	*ff*
tt	*tt*	*ff*	*ff*	*tt*	*tt*	*tt*	*tt*
tt	*tt*	*ff*	*tt*	*tt*	*ff*	*ff*	*ff*
tt	*tt*	*tt*	*ff*	*tt*	*ff*	*tt*	*tt*
tt	*tt*	*tt*	*tt*	*tt*	*tt*	*ff*	*ff*

Phew!

It is enough to look at all cases where η and θ are true. There are 4 cases. In all of these cases, ε is true, while ζ take both values. So A have to leave and we can't say anything about B.